Homework Problems
Physics 417/517
Due December 01, 2009

1. What is the critical (angular) frequency ω_c of Argonne National Laboratory's Advanced Photon Source (APS)? The usual electron beam energy is 7 GeV and the bending radius is 38.96 m. What is the wavelength of the photons at the critical frequency?

2. Using the FEL/undulator resonance condition, and the fact that APS "Undulator-A" designs can operate with K between 0.1 and 2, depending on the pole-gap size, and the undulator period of 2.3 cm, to estimate the range of wavelengths that can be produced from such undulators installed as insertion devices in the APS ring. Suppose one wanted to produce THz (= 10^{15} $\sec^{-1}$) electromagnetic radiation from an undulator with the same period and $K = 1$. How much electron beam energy is needed?

3. Construct a "fish" diagram for $\phi_s = 35^\circ$. At a minimum, show the separatrix, two constant energy curves inside the separatrix, and one constant energy curve outside of the separatrix. Also, display at least 10 points on each curve. It will be easiest to generate the results with a programmable calculator or simple computer code.

4. From $d\tau = dt/\gamma$, and the expression for the Lorentz-invariant power
$$P = -\frac{q^2}{6\pi\varepsilon_0 c}\frac{du^\mu}{d\tau}\frac{du_\mu}{d\tau}$$
show the Lienard expression applies
$$P(t) = \frac{q^2}{6\pi\varepsilon_0 c}\gamma^6\left(\dot{\vec{\beta}}^2 - \left[\vec{\beta}\times\dot{\vec{\beta}}\right]^2\right).$$

5. Two exact integrals involving the modified Bessel function $K_{5/3}$ are
$$\int_0^\infty \xi'^2 K_{5/3}(\xi')\,d\xi' = \frac{16\pi}{9\sqrt{3}}$$
$$\int_0^\infty \xi' K_{5/3}(\xi')\,d\xi' = \frac{5\pi}{3}.$$
From these two formulas show
$$P = \int_0^\infty \frac{dP}{d\omega}d\omega = \frac{\sqrt{3}}{8\pi^2\varepsilon_0}\frac{e^2}{\rho}\omega_c\gamma\int_0^\infty \xi\int_\xi^\infty K_{5/3}(x)\,dx\,d\xi = \frac{e^2 c}{6\pi\varepsilon_0\rho^2}\gamma^4$$
and
$$\langle\hbar\omega\rangle = \frac{8}{15\sqrt{3}}\hbar\omega_c\,.$$

Hint: Apply Fubini's Theorem.

6. Normalize the Gaussian-elliptical phase space distribution

$$\rho(x,x') = A\exp\left(-\left(\gamma x^2 + 2\alpha xx' + \beta x'^2\right)/2\varepsilon\right)$$

assuming $\beta\gamma - \alpha^2 = 1$. Show the statistical average definitions of $\alpha, \beta,$ and γ evaluate to exactly the correct values for this distribution, and $\varepsilon_{rms} = \varepsilon$.